# AMSC/CMSC460 Computational Methods  Spring 2015

**Homework 9, Due on Thursday, April 30, 2015**

**1.** *(Multi-step method)* Consider the following multi-step method

$$y_{n+3} = y_{n+2} + h\left(\frac{23}{12}f(x_{n+2}, y_{n+2}) - \frac{4}{3}f(x_{n+1}, y_{n+1}) + \frac{5}{12}f(x_n, y_n)\right).$$

It is known as one method in *Adams-Bashforth* family.

**a)**. Is this method explicit or implicit?

**b)**. Read and run the Matlab script `multistep.m`. It is a test on the method with an initial value problem:

$$y' = -y, \quad y(0) = 1.$$

The error at $x = 1$ is given numerically. What is the numerical order of accuracy of the method?

*Remark: note the true solution of the system is $y(x) = e^{-x}$. In the code, $y_2 = y(h)$ and $y_3 = y(2h)$ are generated with the exact value of the solution.*

**c)**. Modify the code and generate $y_2$ and $y_3$ using forward Euler method. What numerical order of accuracy do you observe?

**d)**. Which method can we use to get $y_2$ and $y_3$ so that there is no loss of accuracy? Propose one and test it with the code.

**2.** *(Region of absolute stability)* Consider the initial value problem

$$\begin{cases} y' = \lambda y \\ y(0) = y_0, \end{cases}$$

where $\lambda$ is a negative real number.

**a)**. Write down forward Euler scheme explicitly. Find the region of absolute stability in terms of $z = \lambda h$.

**b)**. Let $\lambda = -10$. What is the condition on stepsize $h$ to guarantee absolute stability for forward Euler scheme?

(You can try to test it in Matlab, and it is easy to discover the instability when $h$ is too big. The Matlab implementation is not required to submit.)

**c)**. Find the region of absolute stability for backward Euler scheme. What condition on $h$ ensures absolute stability?