AMSC/CMSC460 Computational Methods **Spring 2015**

Homework 4, Due on Thursday, March 12, 2014

1. *(Polynomial interpolation)* Let f be a function with the following point values:

x_i	1	2	3	4
f_i	-5	-3	2	4

$P_3(x)$ is a polynomial of degree 3 that interpolates f at points $\{x_i\}$.

a). Express P_3 in the Lagrange form $P_3(x) = \sum_i f_i L_i(x)$.

b). Express P_3 in the Newton's representation. Also, find a polynomail $P_2(x)$ of degree 2 that interpolates f at $x = 2, 3, 4$. (You do not have to start over thanks to the Newton's representation.)

c). Express $P_3(x) = \sum_i a_i x^i$. Find the coefficients $\{a_i\}$ by forming the Vandermonde matrix and solving the linear system (you can use Matlab to help solving the 4-by-4 system).

d). Check the polynomials you get from a), b) and c) are the same.

2. *(Runge's phenomenon)* Consider function

$$f(x) = \frac{1}{1 + 25x^2}, \quad x \in [-1, 1].$$

Let $P_n(x)$ be the polynomial of degree n that interpolates f at equally distributed points

$$x_i = \frac{2i}{n} - 1, \quad i \in \{0, 1, \cdots, n\}.$$

a). Read through the code `runge.m`. Plot P_5, P_{10}, P_{20} and compare them with f. What do you observe? Do polynomial approximations have good performance as n becomes larger?

b). Instead of sampling from equally distributed points, we sample from Chebyshev nodes

$$z_i = \cos\left(\frac{2i + 1}{2(n + 1)}\pi\right), \quad i \in \{0, 1, \cdots, n\}.$$

Let Q_n be the polynomial of degree n that interpolates f at $\{z_i\}$. Modify the code, plot Q_5, Q_{10}, Q_{20} and compare with f. What do you observe?

(No need to submit the code. Print out the graphs and write down your observations and comments.)