AMSC/CMSC460 Computational Methods Spring 2015

Homework 2, Due on Tuesday, February 17, 2014

1. *(Matrix norms)*

a). Finish exercise 2.7 in Suli's book: prove that

$$\|A\|_1 = \max_{j=1}^{n} \sum_{i=1}^{n} |a_{ij}|.$$

b). Finish exercise 2.8 in Suli's book: show equivalence of vector norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$, as well as matrix norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$.

2. *(Hilbert matrix)* Hilbert matrix H is defined entry-wise by $h_{ij} = \dfrac{1}{i+j-1}$.

a). Write down a 5-by-5 Hilbert matrix H.

b). Compute $\|H\|_1$ and $\|H\|_\infty$ by hand.

c). Use matlab to calculate condition number of H: κ_1, κ_2 and κ_∞. Try `cond`.

d). Compute condition numbers for 100-by-100 Hilbert matrix. Try `hilb` to create the matrix. Is the matrix well-conditioned or ill-conditioned?

3. *(Linear regression)* Linear regression is widely used in statistics. The idea is to find a line which best fits the data. As an example, we consider the following data set.

X	1	2	4	5
Y	3	5	8	10

The goal is to find a line $y = \alpha + \beta x$ which best fits the data. More precisely, we woule like to have the least-square error:

$$\min_{\alpha,\beta} \sum_{i=1}^{4} |Y_i - (\alpha + \beta X_i)|^2.$$

a). Write the minimization problem in the standard form $\min_{\mathbf{x}} \|A\mathbf{x} - b\|_2^2$. What is $A, b, \mathbf{x}$ in this case?

b). Solve the minimization problem and generate the best line.

4. *(QR factorization)* Finish exercise 2.15 in Suli's book.