AMSC/CMSC460 Computational Methods **Fall 2014**

Homework 3, Due on Thursday, September 25, 2014

1. *(Newton's method)* Write a Matlab code for Newton's iteration method $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$. (You can get a reference code in Moler's book, pp. 119.)

a). Find a root of the zeroth-order Bessel function $J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+1)} \left(\frac{x}{2}\right)^{2m}$. You can directly use Matlab function `besselj` to represent J_0 and J_0' as below:

```
f = @(x)besselj(0,x);
fprime = @(x) - besselj(1,x);
```

as Bessel function has the property $J_0'(x) = -J_1(x)$. Test with $x_0 = 4$ and $x_0 = 5$.

b). Find a root for $f(x) = x^2 - 187$. Start with $x_0 = 187$. Use the numerical result to verify that Newton's method has quadratic convergence rate:

Compute $\dfrac{e_{k+1}}{e_k^2}$ and check if this number converges to a positive constant. What number do you expect (calculate the number by hand)? Does it supported by the numerical result?

c). Find a root for $f(x) = (x^2 - 187)^2$. Start with $x_0 = 187$. Note the root is still $\sqrt{187}$. Check $\dfrac{e_{k+1}}{e_k^2}$ to see what happened. Expain why Newton's method only provides a linear convergence. Compute $\dfrac{e_{k+1}}{e_k}$ to verify that the convergence is linear. Calculate $\lim_{k\to\infty} \dfrac{e_{k+1}}{e_k}$, and check it with the numerical result.

2. *(Square root method)* Consider the following iteration procedure

$$x_{k+1} = x_k - sgn(f'(x_k)) \frac{f(x_k)}{\sqrt{f'(x_k)^2 - f(x_k)f''(x_k)}}.$$

This method is called *square root method*. Prove that this method has a local cubic convergence if $f'(x_*)^2 - f(x_*)f''(x_*) > 0$.

3. *(Secant method)*

a). Prove that secant method has a rate of convergence $q = \frac{1}{2}(1 + \sqrt{5})$. You can follow the storyline of excercise 1.10 in Suli's book.

b). Write a Matlab code on secant method and test $f(x) = x^2 - 187$ with $x_0 = 187$, $x_1 = 185$. Check the numerical convergence rate.