AMSC/CMSC460 Computational Methods Fall 2014

Homework 2, Due on Tuesday, September 16, 2014

1. *(Matrix norms)*

a). Finish exercise 2.7 in Suli's book: prove that

$$\|A\|_1 = \max_{j=1}^{n} \sum_{i=1}^{n} |a_{ij}|.$$

b). Finish exercise 2.8 in Suli's book: show equivalence of vector norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$, as well as matrix norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$.

2. *(Gauss elimination with pivoting)* Let A be a 3-by-3 matrix given as

$$A = \begin{pmatrix} 3 & 1 & 6 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}.$$

a). Find a lower triangular matrix L, an upper triangular matrix U and a permutation matrix P such that $PA = LU$.

b). Use matlab code $[L, U, P] = \texttt{lu}(A)$ to check your answer.

c). Run matlab code $[L2, U] = \texttt{lu}(A)$. What is $L2$? Derive $L2$ by hand.

d). Solve the linear system $Ax = b$ by hand, with $b = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$.

3. *(Matlab implementation of Gauss elimination)*

a). Read through the attached Matlab code on function `mylu`. Test with A, b given in problem 2. Find an LU decomposition of A, and the solution of $Ax = b$ from the code.

b). Modify the code by adding pivoting to the Gauss elimination procedure. Write a Matlab function $[x, LU, p] = \texttt{mylup}(A, b)$, where the input A is an n-by-n matrix, b is an n-vector, and the output x is the solution of $Ax = b$, LU is the matrix containing the information of LU decomposition, and p is the permutation vector. Do NOT use Matlab functions like backslash, `lu`, etc.

c). Test your code in b) on A, b given in problem 2. Check your solution with Matlab integrated functions:

- For x, check with $A\backslash b$.
- For LU, check with $\texttt{lu}(A)$.
- For p, check with the p in the output of $[L, U, p] = \texttt{lu}(A, \texttt{'vector'})$.