CAAM/MATH423 Partial Differential Equations I **Fall 2015**

Homework 3, Due on Tuesday, September 15, 2015

1. *(Alternative way of deriving d'Alembert's formula)* Finish problem 19 in section 2.5 of Evans book.

a). Show the general solution of the PDE $u_{xy} = 0$ is

$$u(x, y) = F(x) + G(y)$$

for arbitrary functions F, G.

b). Using the change of variables $\xi = x+t$, $\eta = x-t$, show $u_{tt} - u_{xx} = 0$ if and only if $u_{\xi\eta} = 0$.

c). Use (a) and (b) to rederive d'Alembert's formula.

d). Under what conditions on the initial data g, h is the solution u a right-moving wave? A left-moving wave?

2. *(Linear homogenous hyperbolic equation)* Solve the initial value problem:

$$u_{xx} - 3u_{xt} - 4u_{tt} = 0, \quad u(x, t = 0) = x^2, \quad u_t(x, t = 0) = e^x.$$

Hint: You can factor the operator $\partial^2_{xx} - 3\partial^2_{xt} - 4\partial^2_{tt}$ *and derive a formula similar to the derivation of d'Alembert's formula.*

3. *(Energy estimate and uniqueness)* Consider the following equation characterizing vibrating strings with air resistance

$$u_{tt} - c^2 u_{xx} + r u_t = 0, \quad u(x, t = 0) = g(x), \quad u_t(x, t = 0) = h(x).$$

where c is the wave speed, and $r > 0$ is the resistance coefficient.

a). Define *energy* $E(t) = \int_{\mathbb{R}} \left(u_t^2(x,t) + c^2 u_x^2(x,t)\right) dx$. Show that $E(t)$ decays in time.

b). Prove that the classical solution of the initial value problem (if exists,) is unique.