MATH322 Introduction to Mathematical Analysis II **Spring 2016**

Homework 1, Due on Wednesday, January 20, 2016

1. *(Compact sets)* Let X be a metric space in which every infinite subset has a limit point. Prove that X is compact. *Hint: see a series of hints in page 45 of Rudin's book.*

2. *(Convergence of power series)* Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^{2n}}{n \cdot 5^{n+2}} x^{n+1}$, where $x \in \mathbb{R}$. (Be careful with the endpoints.)

3. *(Uniform continuity)* Let X, Y, Z be metric spaces. $f : X \to Y$ and $g : Y \to Z$ are uniformly continuous mappings. Prove that $g \circ f$ is uniformly continuous from X to Z. Here $\circ$ represents composition, i.e. $g \circ f(x) = g(f(x))$.

4. *(Sequence of functions)* Find an example of a sequence of continuous functions $\{f_n\}$, such that the pointwise limit function f exists but not continuous.