MATH302 Elements of Analysis **Spring 2018**

Homework 6, Due on Tuesday, March 27, 2018

For all the proofs, write *in details* using standard mathematical language.

1. *(The union of open sets)*

a). Prove Theorem 8.3 in the textbook: the union of any collection of open sets is open.

b). (No need to submit) Read the proof of the Theorem 8.11 in the textbook: S is open if and only if there is a countable collection of mutually disjoint open intervals $\{U_1, U_2, \cdots\}$ such that $S = \cup_n U_n$.

2. Fill the boxes $\square$ with $\subseteq$, $\supseteq$ or $=$, and the prove the statements. Let $f : A \to B$ be a mapping.

a). Let $A_1, A_2 \subseteq A$. Then, $f(A_1 \cup A_2) \square f(A_1) \cup f(A_2)$.

b). Let $B_1, B_2 \subseteq B$. Then, $f^{-1}(B_1 \cap B_2) \square f^{-1}(B_1) \cap f^{-1}(B_2)$.

c). Let $B_1, B_2 \subseteq B$. Then, $f^{-1}(B_1 \cup B_2) \square f^{-1}(B_1) \cup f^{-1}(B_2)$. (No need to prove this one)

3. *(Squeeze theorem)* Let f, g, h be functions in $\mathbb{R} \to \mathbb{R}$. Let $a \in \mathbb{R}$. Suppose there exists a $\delta_0 > 0$ such that $f(x) \leq g(x) \leq h(x)$ in a δ_0-neighberbood of a, namely for any $x \in (a - \delta_0, a + \delta_0)$. Moreover, f and h are continuous at point a, and $f(a) = h(a)$. Then, g is also continuous at a.