

## Homework 1, Due on Thursday, January 25, 2016

For all the proofs, write *in details* using standard mathematical language.

**1.** Show that  $(A \Leftrightarrow B) \Leftrightarrow (\neg A \Leftrightarrow \neg B)$ .

*Hint: use the definition of “ $\Leftrightarrow$ ” (logically equivalent) and the truth table.*

**2.** Prove the following statement:

$$\forall x > 0 \exists y > 0 (y > x \text{ and } y < 2x).$$

**3.** Prove that if a function  $f$  satisfies the Hölder condition with power  $1/2$ :

$$\exists C > 0 \forall x \in \mathbb{R} \forall y \in \mathbb{R} (|f(x) - f(y)| \leq C|x - y|^{1/2}),$$

then  $f$  is uniformly continuous, namely

$$\forall x \in \mathbb{R} \forall \epsilon > 0 \exists \delta > 0 \forall y (|x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon).$$

**4.** Let  $S = \{x : x = 5n, n = 1, 2, \dots\}$  and  $T = \{x : x = 10n + 5, n = 1, 2, \dots\}$ .

a). Show that  $T \subseteq S$ .

b). Show that  $S$  and  $T$  have the same cardinality.

**5.** Show that if  $A$  and  $B$  are in one-to-one correspondence,  $B$  and  $C$  are also in one-to-one correspondence. Then,  $A$  and  $C$  are in one-to-one correspondence.