MATH141(0332/0342) Calculus II Fall 2009

Worksheet 12, Complex Numbers Name: _____

1. (*8 points*) Compute the following: (You anwser should be in $a+bi$ form.)

(1) $e^{1-i\cdot\frac{\pi}{3}}$

(2) $\dfrac{(5+i)(7-i)}{4+3i}$

(3) $\left|(6+i)e^{i\cdot\frac{5}{3}}\right|$

(4) $(\sqrt{3}+i)^{10}$

2. (*4 points*) (1) Find all complex roots of the equation $z^5+1=0$ in $a+bi$ form.

(2) Find all square roots of -3+4i in both $re^{i\theta}$ form and $a+bi$ form.

3. (*4 points*) Find for which complex value of z does the series converges.

(1) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n-1)!}z^n$

(2) $\displaystyle\sum_{n=1}^{\infty}\frac{n^n(2n)!}{(3n+2)!}z^n$

4. (*4 points*) Prove the following equality.

$$1+r\cos(\theta)+r^2\cos(2\theta)+\cdots+r^n\cos(n\theta)+\cdots=\frac{1-r\cos(\theta)}{1-2r\cos(\theta)+r^2}$$

Hint: Notice that $\cos(k\theta)$ is the real part of $e^{i(k\theta)}$. So the left hand side can be written as the real part of a complex geometric series. The same formula for geometric series will work.