MATH141(0332) Calculus II

Quiz 8, November 6 - 11, 2008 Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. This is a take-home quiz. Please hand in your solution in the discussion on November 11(Tuesday).
This quiz is worth 12 points. 10 of them will be counted to your final score.

1. (*3 points*) Find the Taylor polynomial of the following f for the given value $n = 2$, around the point $x = 0$.

$$f(x) = \begin{cases} \frac{sin(x)}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Hint: $f'(0) = \lim_{x \to 0} f'(x)$, if the limit exists. To calculate this, you might use L'Hopital's rule.

2. (*5 points*) Let $\{a_n\}_{n=1}^{\infty}$ be the sequence $\sqrt{6}, \sqrt{6+\sqrt{6}}, \sqrt{6+\sqrt{6+\sqrt{6}}}, ...,$ where in general, $a_{n+1} = \sqrt{6+a_n}$.
(1) Show that for every positie integer n, $0 < a_n < 3$. (Notice:$\sqrt{6} < 3$)

(2) Show that for every positie integer n, $a_n \leq a_{n+1}$.
Hint: $a_{n+1} - a_n = a_{n+1} - (a_{n+1}^2 - 6)$. You might find the result of (1) is useful to prove (2).

(3) Show that $\{a_n\}$ converges, and find the value of $\lim_{n \to \infty} a_n$.

3. (*4 points*) Determine whether the following infinite sum converges or diverges. If it converges, find its sum. (Use the back side of the paper to finish this problem.)

$$(1)\ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n+1}}{5^{n-2}} \qquad (2)\ \sum_{n=1}^{\infty} \frac{3n^3 - n}{\sqrt{n+1}}$$