

STAT100 Elementary Statistics and Probability Summer II 2014

Quiz 7, Tuesday, August 4, 2014

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Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. You are allowed to calculator for basic calculation in this quiz. You have 15 minutes to take this 10 point quiz.

1. (6 points) NBA player LeBron James has a 75% free throw percentage in his career. What is the approximate probability that he made at least 200 free throws in 300 attempts?

Let $X =$ number of free throws made in 300 attempts.

$$X \sim \text{Binom}(300, .75).$$

$$\left(\begin{array}{l} p = .75 \quad \mu = np = 225 \\ q = .25 \quad \sigma^2 = npq = 56.25 \end{array} \right)$$

We would like to find $P(X \geq 200)$.

As $300 \times .75 = 225 > 15$, $300 \times .25 = 75 > 15$, X obeys a normal distribution approximately. $X \overset{\text{approx.}}{\sim} N(225, 56.25)$.

$$\begin{aligned} P(X \geq 199.5) &= P\left(\frac{X - 225}{7.5} \geq \frac{199.5 - 225}{7.5}\right) = P(Z \geq -3.4) \\ &= 1 - \Phi(-3.4) = \Phi(3.4) = .9997. \end{aligned}$$

2. (4 points) Suppose X_1, \dots, X_{20} are independent identical normal distributions with mean 4 and variance 5. \bar{X} is the sample mean. Find $P(4 \leq \bar{X} \leq 5)$.

X_1, \dots, X_{20} are i.i.d. $N(4, 5)$.

$$\Rightarrow \bar{X} \sim N\left(4, \frac{5}{20}\right) = N(4, .25).$$

$$\begin{aligned} \text{So, } P(4 \leq \bar{X} \leq 5) &= P\left(\frac{4-4}{.5} \leq \frac{\bar{X}-4}{.5} \leq \frac{5-4}{.5}\right) \\ &= P(0 \leq Z \leq 2) = \Phi(2) - \Phi(0) \\ &= .9772 - .5 = .4772. \end{aligned}$$